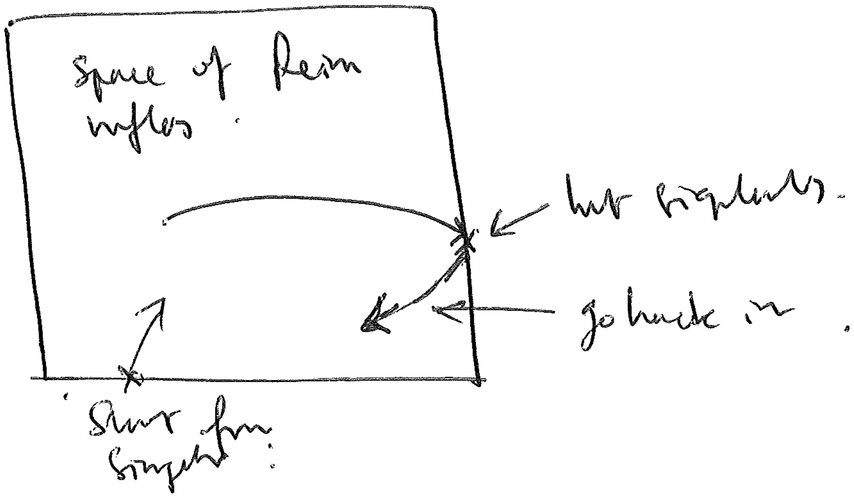
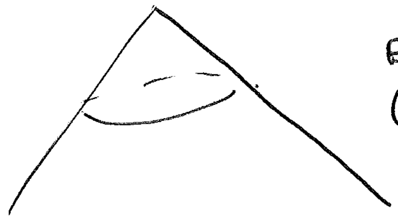
Juillet.: Examples in relation with a metric flow. 29/08/2016.



Problem: going into space of Reim mfld not unique.

Gigli-Mantegazza: $(X,d) \xrightarrow{\tau} (X,d^\tau)$.

Three examples (I). $g = dr^2 + r^2 d\theta^2$.
$\mathbb{R}^+ \times \mathbb{R}/\alpha\mathbb{Z}$. $\alpha \leq 2\pi$.



Euclidean Cone. (Alexo. Space).

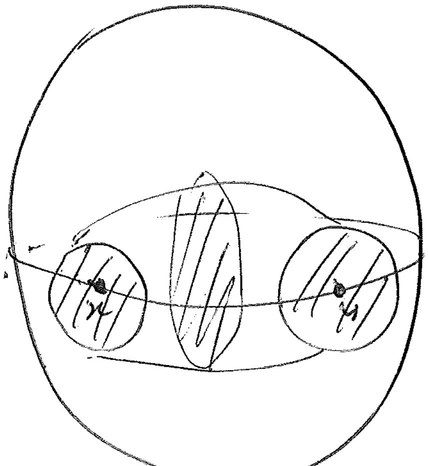
(II). $N(y-x) = \inf_{\substack{\gamma(0)=x \\ \gamma(1)=y}} \int N(\dot\gamma(t))\,dt$. Vspace $(\mathbb{R}^2, N)$.

(Finsler geometry).

(III). Heisenberg group: $\mathbb{R}^2 \times \mathbb{R}$, Sub-Reim geometry

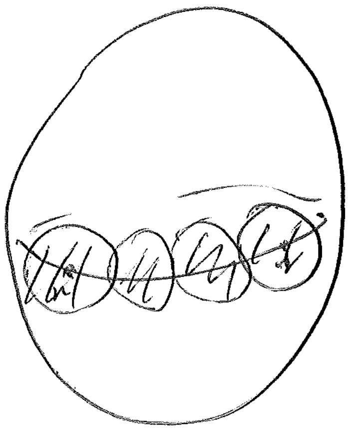
Riem mfld example for GM flow.

$X = \mathbb{S}^2$, μ_x^τ ball of centre x, radius τ.



for quasi-metric $\hat{d}^\tau$.

$\Downarrow$ lumpersicize d^τ.



$t \longmapsto \mu_{x_t}$.

$$\frac{d}{dt}\mu_{x_t} + \mathrm{div}(v_t \mu_{x_t}) = 0.$$

$$\text{Speed}(x_t) \le \sqrt{\int \|v_t\|^2 \, d\mu_{x_t}}$$

equality: v_t is gradient $v_t = \nabla \varphi_t$.

Thm (GM) (M,g) compact Riem mfld, then (M, d^τ) Riem (given by g^τ). and

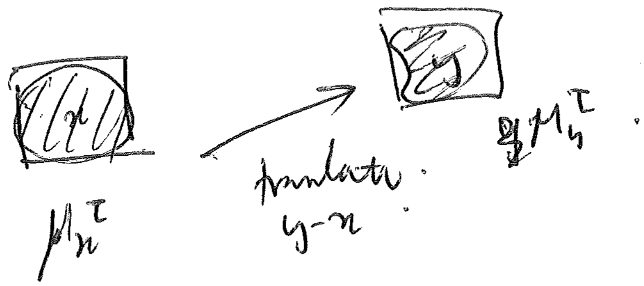
$$\left.\frac{d}{d\tau}\right|_{\tau=0} g^\tau(\dot\gamma(s), \dot\gamma(s)) = -2\,\mathrm{Ric}_{g_0}(\dot\gamma(s), \dot\gamma(s))$$

s-a.e., γ g-geodesic.

Remarks:

- $\dot\gamma(t)$ a.e. t, Not for any $v \in TM$.
- Not a semi-group $(d^\tau)^{\tau'} \neq d^{\tau+\tau'}$.
- Bandera–Lakzian–Munn.

Space 1 . $(\mathbb{R}^2, N)$. Finsler .



$$\tilde{d}(x,y) = W(\mu_x^\tau, \mu_y^\tau) = \sqrt{\int N(p-q)^2 \, d\pi(p,q)}$$

$$\geq \sqrt{N^2 \int p-q \, d\pi(p,q)} \quad \text{Jensen's}$$

$$= N(y-x).$$

$$N(y-x) \geq W(\mu_x^\tau, \mu_y^\tau)$$

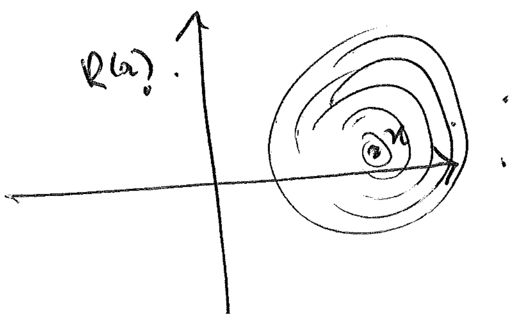
$$\Rightarrow d^\tau = \tilde{d}^\tau = N(-) = d.$$

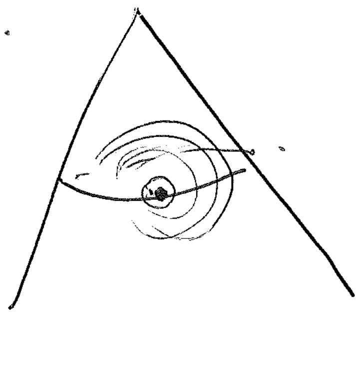
Space 2 . Euclidean Cone.

$$\alpha = \frac{2\pi}{m} \quad (m=2 \text{ or } m=4).$$

def. $R : z \in \mathbb{C} \longmapsto z e^{i\frac{2\pi}{m}}$.
$\quad\quad\quad\quad \mathbb{R}^2$.

Fact. $\text{Cone}\left(\frac{2\pi}{m}\right) = \mathbb{R}^2 / \langle R \rangle$.





Space 3 . $\mathbb{R}^2 \times \mathbb{R}$. Heisenberg . Reim dist

Carnot – Carathéodory .

$$g_\varepsilon = \begin{pmatrix} 1 & & \\ & 1 & \\ & & \frac{1}{\varepsilon} \end{pmatrix} \xrightarrow{\varepsilon \to 0} g_{cc} \begin{pmatrix} 1 & & \\ & 1 & \\ & & +\infty \end{pmatrix}.$$

g_{cc} given by

$$X = \partial_x - \tfrac{1}{2} y\, \partial_z .$$
$$Y = \partial_y + \tfrac{1}{2} x\, \partial_z .$$
$$Z = \partial_z .$$

Th^m. $\exists\, K > 1 .\ \exists\, k > 0 .\ \forall \tau > 0,\ d^\tau_{cc}$ is Reimannian . metr

$$g_\tau = \begin{pmatrix} k & & \\ & k & \\ & & k/\tau \end{pmatrix}.$$