Takashi Shioya. 03/03/2015.

Study of spaces $X_n \xrightarrow{n\to\infty} X$, metrics + measures.

esp when $\dim X_n \to +\infty$

Main Thm: (r_n): sequence of pos. real numbers.

(I) $r_n/\sqrt{n} \to 0 \Leftrightarrow S^n(r_n) \to *$ (one pt. space)

(II) $r_n/\sqrt{n} \to +\infty \Leftrightarrow S^n(r_n) \to \infty$ Concentration. dissipation.

(III) $r_n/\sqrt{n} \to \lambda \in (0,+\infty) \Leftrightarrow S^n(r_n) \to \Pi^\infty_{\lambda^2}$

$\Pi^\infty_{\lambda^2} = (\mathbb{R}^\infty, \|\cdot\|_2, \gamma^\infty_{\lambda^2})$ ∞-dim Gaussian.

l^2 norm, may take $+\infty$ as value.

$\gamma^\infty_{\lambda^2}$: centred Gaussian measure with variance λ^2.

Ideas: diam & obs diam (observable diam).

X mm, $k>0$.

$\operatorname{diam}(\mu_X, -k) := \inf\{\operatorname{diam} A : A \subset X, \mu_X(A) \geq 1-k\}$.

$\operatorname{obs\,diam}(X,-k) := \sup_{f: X\to\mathbb{R}\ 1\text{-Lip}} \operatorname{diam}(f_*\mu_X; -k)$.

$f_*\mu_X(A) = \mu_X(f^{-1}(A))$, $A \subset \mathbb{R}$.

obsdiam: monotone increasing in k.

$= 0$ for $k \geq 1$.

$\operatorname{obsdiam}(tX; -k) = t\operatorname{obsdiam}(X;-k)$.

(Lévy-Milman) : obsdiam$(X_n; -k) \to 0 \cdot \forall k > 0$.

$\{X_n\}$ Lévy family $X_n \to \circledast$ ← point.

"Any 1-Lip func $f_n : X_n \to \mathbb{R}$ is almost constant for large n."