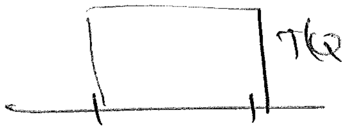
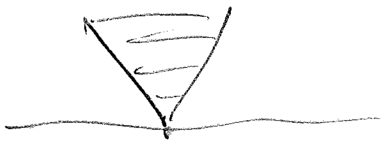
Jill Pipher — Ahe [illegible] of Elliptic Δ. 27/07/2015.

Parabolic Measure.

$Lu = 0$ or $Lu = \partial_t u$, $L = -\operatorname{div} A\nabla$. ← $\mathbb{R}$, but non-symm.

Recall Carleson:  $T(Q)$. $\mu(T(Q)) \le c|Q|$.

Q

$\Gamma(x)$ = cone 

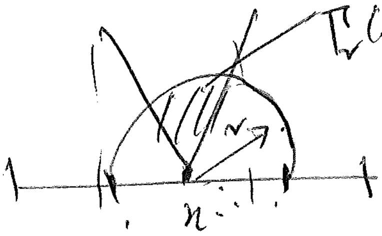
x

$$S(u)(x) = \left(\int_{\Gamma(x)} y^{1-n}|\nabla u(x',y)|^2\, dx'\, dy\right)^{\frac{1}{2}}.$$

$$\int_{\mathbb{R}^{n}} S^2(u) = \int_{\mathbb{R}^{n+1}} y|\nabla u(x',y)|^2 dx'dy = c\int_{\mathbb{R}^n} f^2(x)\,dx.$$

for $\Delta u(x,t) = 0$, $u(x,0) = f(x)$.

$f \in BMO$: $S_r(u)$ integral over $\Gamma_r(x)$.



Dirichlet pb w/ L^p data: $p > 1$. $\Delta u = 0 \in \mathbb{R}^{n+1}_+$; $u(x,0) = f(x) \in L^p$.

in nontangential sense.

$L := -\operatorname{div} A\nabla$, $A(x) \in M_{\mathbb{R}^{n+1}\times\mathbb{R}^{n+1}}$ poss non-symm.

Elliptic pbm. $\lambda|\xi|^2 \le \langle A\xi, \xi\rangle$, $\|A\|_{L^\infty} \le \lambda^{-1}$.

If $Lu = 0$, $u(x,0) = f(x)$, then

$u(X) = \int_{\mathbb{R}^n} f(x)\, d\omega^X(x)$ ← Elliptic measure assoc. with sol^n.

- De Giorgi – Nash – Moser : weak sol^{n}s are cts.
- Harnack principle, interior Hölder, expansion.
- $d\omega^x(x)$ are mutually abs. cts.
- u defined ω..., u^* comparable to weighted max op.
- Solvability of D_p equiv to regularity of weights ω:

  $\|u^*\|_p \leq C\|f\|_p$ iff $d\omega = k\,dx$. and $k \in \underbrace{RH_{p'}}_{\text{Reverse Hölder of order } p'}(dx)$.

- Solvable for $\underline{\text{some}}$ p, iff $d\omega^x \in A_\infty$.

$\underline{\text{Def}^{\text{n}}}$: $A_\infty \ni \omega$.

(I) $\forall \varepsilon \in (0,1)$, $\exists \delta \in (0,1)$. s.t. $\forall Q$ cubes. $E \subset Q$,

$$\omega(E)/\omega(Q) < \delta \Rightarrow |E|/|Q| < \varepsilon.$$

Notion of ε-approximability: $u \in L^\infty$, $\|u\|_\infty \leq 1$.

$\forall \varepsilon > 0$, $\forall Q$ cubes, $\exists\, \varphi_Q \in W^{1,1}(T_Q)$ s.t.

$$\|u - \varphi_Q\|_{?} < \varepsilon$$

and $\int_{T(Q)} |\nabla \varphi(x,y)|^2\, dx\,dy \leq C|Q|$. (?) | Check.

ε-approx $\Rightarrow A_\infty$.

$\underline{\text{Th}^{\text{m}}}$ (Dindos – Kenig – P. 2011).

$\omega \in A_\infty$ iff $\forall u$ $Lu = 0$ with BMO bdy data f,

$$\sup_Q \frac{1}{|Q|} \iint_{T(Q)} y\,|\nabla u(x,y)|^2\, dx\,dy \leq C\,\|f\|^2_{BMO}.$$

<u>Idea of Pf:</u> show (*) iff for Carleson condition of w.

(I) f bdd, $f = 1$ on E, 0 outside $2Q$.

$$w(E)/w(Q) < \frac{1}{|4Q|}\iint_{T(4Q)} y\,|\nabla u(x,y)|^2\,dx\,dy.$$

(II). Jones–Journé construction to find such a function but with $\|f\|_{BMO}$ small.

$$f = \max\{\underbrace{\delta \log(M\chi_E)}_{\text{HL max.}} + 1, 0\}.$$

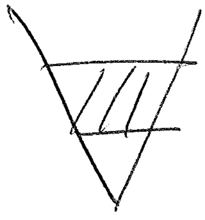
u sol^n to $Lu = 0$ with data f.

Th^m (Kenig–Kirchheim–P–T).
$w \in A_\infty$ iff for $\forall f$ bdd. Carleson cond. holds for u: $\sup_Q \frac{1}{|Q|}\iint_{T(Q)} y\,|\nabla u(x,y)|^2\,dx\,dy \le C$.

↑
Generalizes to parabolic setting!

<u>Pf idea:</u> Good ε_0-cover as Christ cubes for $(\mathbb{R}^d, dw)$.

: control oscillation and chord-



T_m. have $\operatorname{osc}(u) \ge c_0$.

So have $\int_{T_m} y^{1-n} |\nabla u(x,y)|^2\,dx\,dy > c_0$.

- Here have $\int S^2(u) \geq k c_0$. [illegible] can be done.

So. $k c_0 |E| \leq \int S^2(u) \leq \int_Q u |\nabla \text{———}|^2 \leq \delta |Q|$.

So $|E| / |Q|$ can be made small [illegible].

we can find dyadic cond s.t.

$w(E) \to 0$ [illegible] $k \to \infty$.

- Works in Complex coefficient case.

  - but no elliptic means, need Comparison - b/w untangential max & sq. fnctn estimates.