Steve Hofmann. 08/06/2015.

F + M. Reisz. (1916).

$\Omega \subset \mathbb{R}^2$, simply connected.

$\partial\Omega$ rectifiable $\Rightarrow$ $\omega \ll \sigma = \mathcal{H}^1|_{\partial\Omega}$.

(In high dim $\omega \ll \sigma = \mathcal{H}^n|_{\partial\Omega}$).

Lavrentiev (1936). quantitative version.

Bishop-Jones (1990) $E \subset \partial\Omega$, then $\omega|_E \ll \sigma|_E$.

(need connectivity).

Higher Dim: Kenig C.F. quantitative results:

- 77 Dahlberg .. Ω Lipschitz, then $\omega \ll A_\infty(\sigma)$.

for all "surface balls" ($B \cap \partial\Omega = \Delta$).

if $A \subset \Delta$, then

$$\omega(A) \lesssim \left(\frac{\sigma(A)}{\sigma(\Delta)}\right)^\theta \omega(\Delta).$$

- David-Jerison / Semmes '90.

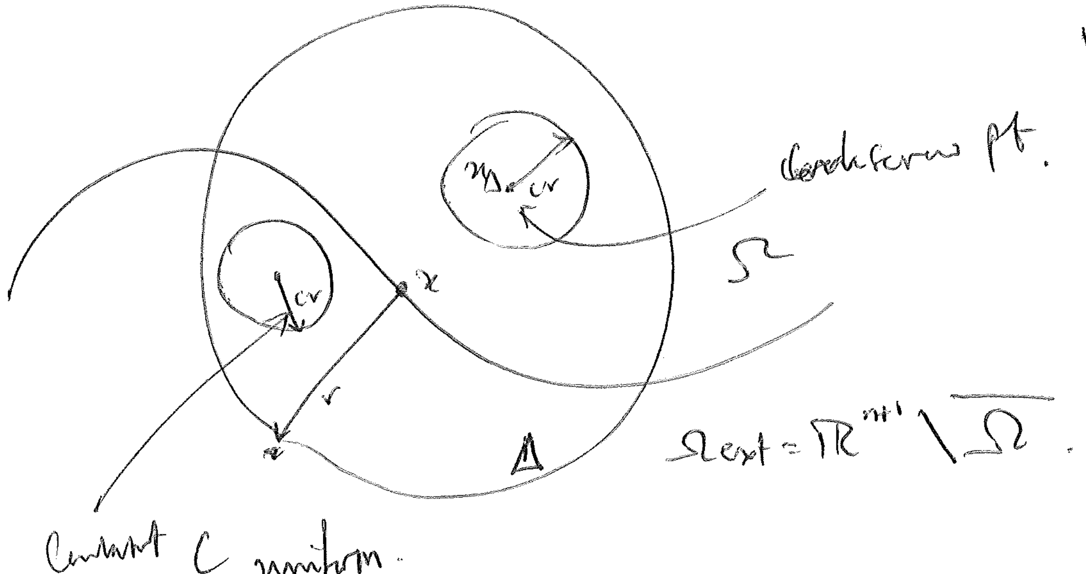
Ω "chord-arc" domain, i.e. Ω NTA

to $\partial\Omega$ ADR, then $\omega \in A_\infty(\sigma)$.

NTA (Jerison-Kenig '91).

Ω Harnack chains (quantitative path conn.).

2 sided Corkscrew (CS) condition.



ADR.
$\sigma(\Delta(q,r)) \simeq r^n$.

method of DJ: Ω has "int. big pieces of Lip sub-dom."

If $A \subset \Delta$; $\sigma(A) \geq (1-\eta)\,\sigma(\Delta)$.

$\therefore$ Dahlberg + max principle.

Bennewitz – Lewis (2004).

2 C-S + $\partial\Omega \in$ ADR $\Rightarrow$ $\omega \in$ weak-$A_\infty(\sigma)$.

Recent results:

Uniform Rectifiability (UR).

David – Semmes. E UR $\iff$ E ADR, dmd, BPLI.

(*) {Sets for which nice SIO's: $L^2 \to L^2$.

$\mathbb{R}^n$

Theorem. Ω uniform Domain (aka 1-sided NTA), $\partial\Omega \in$ ADR. Then TFAE:

(I) $\partial\Omega \in$ ADR

(II) Ω_{ext} has CS (hence Ω NTA).

(III) $w \in A_\infty(\sigma)$.

(IV) $w \in$ weak $A_\infty(\sigma)$.

No connectivity (Recall Ω Harnack chain is a connectivity condition.)

Th^m 1. (S. Bortz, S. H).

Sps $E \subset \mathbb{R}^{n+1}$, n-dim UR. Then E has BP. (σ-dimension)

Consequently E has BP ($w \in$ weak $-A_\infty$).

Th^m 2. (S.H. P. Le, Martell, Nyström).

Sps $\Omega \subset \mathbb{R}^{n+1}$, $\partial\Omega \in$ ADR, $w \in$ weak$-A_\infty$, then $\partial\Omega \in$ UR.

Corollary. Sps $E \in$ ADR. Then ~~E UR~~ E UR iff E has BP ($w \in$ weak$-A_\infty$).

Context Th^m 2: large const. and pt. version to Abh. Coif.,

Jensen, Kenig-Toro.

$$w \in A_\infty(\sigma) \text{ "}\Longleftrightarrow\text{" } \log\left(\frac{dw}{d\sigma}\right) \in BMO.$$

Rmk. p-harmonic version as well

Thm 1: "extrapolation of Carleson measures".

Thm 2: ~~Stopping time~~

Precondition Lewis-Vogel, $\omega \in ADR$, $\frac{d\omega}{d\sigma} \approx 1$.

Step 0: Stopping time.

$\omega \in$ weak-A_∞ $\Rightarrow$ "construct amplified sawtooth" corresponding to a given cube Q_0.

For stopping time Q, $\frac{\omega(Q)}{\sigma(Q)} \approx 1$

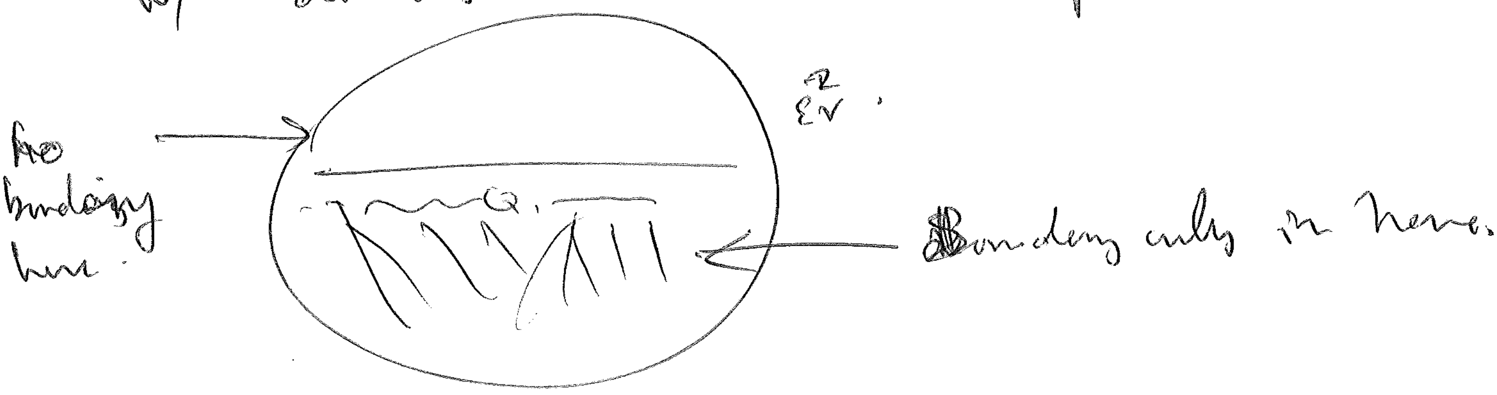
and follow Lewis-Vogel in Q.

Step 1: Follow LV $\Rightarrow$ in ample subregion, have "flatness".

LV flatness "WEC" $\Leftrightarrow$ "BAUP".

But no doubling in this situation, so LV only takes you so far.

w/o doubling "WHSA" weak half space approx.



Step 2: (Last) WHSA $\Rightarrow$ UR,