Knopf . Gflows & Friends . 05/03/2015 .

- Singularities are inevitable for large classes of geometric heat flows .

- Common method of analysis: parabolic dilations .
  & use compactness to get subsequential convergence .
- Sometimes, there is a classification .
- Independence of subsequence? $\longrightarrow$ asymptotic analysis .

<u>Folklore Conjecture</u>: Singularities of geometric heat flows acquire extra symmetries asymptotically .

$\rightsquigarrow$ understood as quasi-isometries .

Singularity solutions are <u>ancient solutions</u> . and
Conjecture: self-similarity .

$\rightarrow$ Generalized fixed points in moduli space .
"maximally" diffused." $\longrightarrow$ should have large symmetry groups .

- $S^1 \hookrightarrow S^3 \twoheadrightarrow S^2$ Hopf fibration .

$$\tilde{G} = f^2\, \omega^1 \otimes \omega^1 + g^2\, \omega^2 \otimes \omega^2 + h^2\, \omega^3 \otimes \omega^3$$

let $h = g$ send $f \to 0$ GH converges to S^2.

3

$(S^1 \times S^3, G(s))$ when $f = f(s,t)$, $g = g(s,t) = h$; warped Berger solution to Ricci-flow and $+ (ds)^2$.

See Isenberg-K-Šešum 2014.

MCF Singularities: { Gang-K-Sigal 2011. Gang-K 2013

$(M, g(t))$ complete, noncompact 2-dim C^3 close. to std neck.

1. singular $T < \infty$.
2. Asymptotically rot. sym. in spacetime nbhd.
3. strongly precise asymptotic profile.
4. uniqueness from given flow.

$\to$ Quantum harmonic oscillator

$$h_a = \partial_y^2 - ay\,\partial_y - 2a.$$

appears in much of asymptotic analysis,

look at $L_{a(\tau)}$ operator - closure in $L^2(\underbrace{e^{-}\,d\sigma}_{\text{Gaussian}})$

is self-adjoint & pt. spectrum.

Unstable, but eigenvalues corresponds to coordinate instabilities