MCF w/ ∂M. B. White. 05/03/2015.

$M^k \subset \mathbb{R}^n$ with ∂M.

on $M \setminus \partial M$, $(\text{velocity})^\perp = H$.

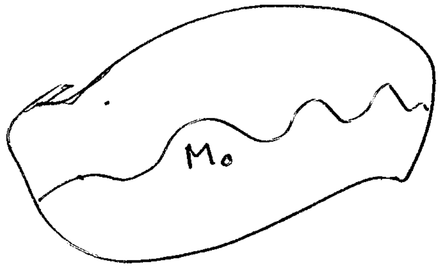
<u>Given</u>. $\partial M_t = \Gamma(t)$.

Can assume $\partial M_t = \Gamma$ (fixed).

Short time existence: PDE

Long time: Ilmanen's elliptic regularisation.

<u>Thm 1</u>:



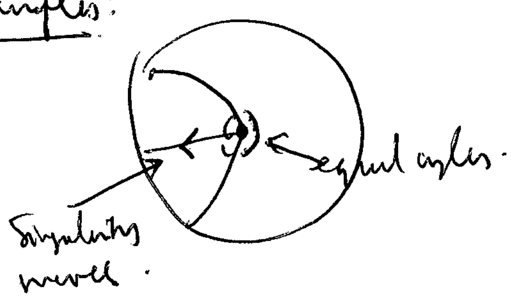
(n-dim hypersurface in $\mathbb{R}^{n+1}$ or Riem mfld).

$$\left\{ \begin{array}{l} M_0 \subset \partial\Omega. \\ \Gamma(t) \subset \partial\Omega \ \forall t. \\ \overline{\Omega} \text{ mean convex}. \end{array} \right.$$

$\Rightarrow \forall t \geq 0$, M_t is regular <u>near</u> $\partial M_t = \Gamma_t$.

<u>Rmk</u> Curve, you get singularities in the interior, but point: they can't move to ∂M.

<u>Counterexamples</u>:



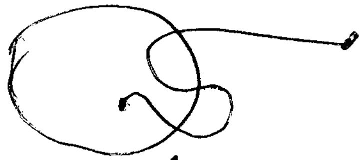
Problem: key hypothesis left out, ie:
what is MCF for possibly singular M_t.

On interior: Brakke flow.

(2) $\partial M_t = \Pi$.

Def^n of $\partial M = \Pi$:

(1) Homological: $\partial M \cong \Pi$ (flat chains mod 2).

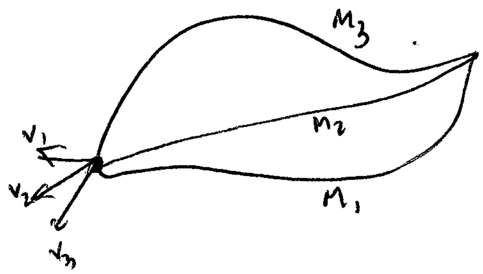


intersects odd times.

(2) Div th^m:

$$\int_M \operatorname{div}_M X \, dA = -\int_M H \cdot X \, dA + \int_{\partial M} X \cdot \nu \, d\mathcal{H}^{n-1}.$$

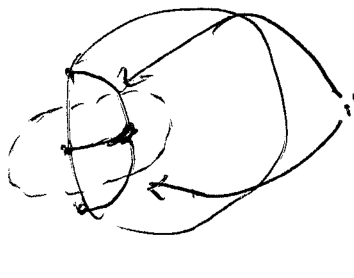
Require $\|\nu\| \leq 1$.



$\nu = \nu_1 + \nu_2 + \nu_3$.

Limits preserve (1) + (2).

Now, 1-d counterexample configuration violates Homology.

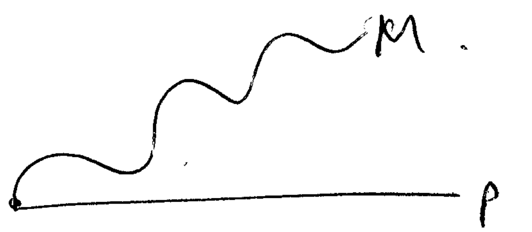


intersects odd number of times

$\underline{\text{Th}^{m}}$. $M^2 \subset \mathbb{R}^3$, self similar, $\partial M = L$, line through 0

M is disjoint union from some half plane P with $\partial P = L$.

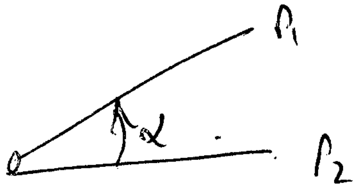
$\Rightarrow M$ = Finite union of half planes. | $g_{ij} = e^{\frac{|x|^2}{4}} \delta_{ij}$.



Cone angle theorem says you can rotate P a little without hitting something at ∞. But P touches M and minimality $\Rightarrow$ M contains plane.

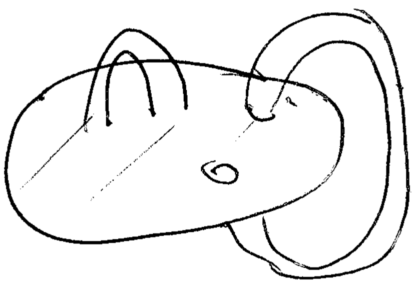
Take out plane & repeat.

<u>Rmk</u>: Noncpt, need cone th^{m} says:

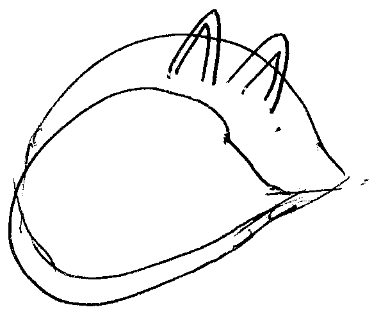


This is a cone $\Rightarrow$ $\exists$ definite angle $\alpha > 0$

<u>Existence of ∂ sings:</u>



"Generalised disk".



"Generalised mob strip."

Lemma. $\exists$ knot $\Gamma \hookrightarrow \mathbb{R}^3$.

$TC(\Gamma) < 4\pi$.

Γ bounds at least one GMS.

No GMS are local min of area.

Thm. Let M be a minimal GMS with $\partial M = \Gamma$. (as in Lemma)

$\exists$ MCF. $\mathbb{R} \ni t \longmapsto M_t$. (external!)

$M_t \longrightarrow M$ smoothly as $t \to -\infty$.

$M_t \longrightarrow \tilde{M}$ smoothly $t \to \infty$.