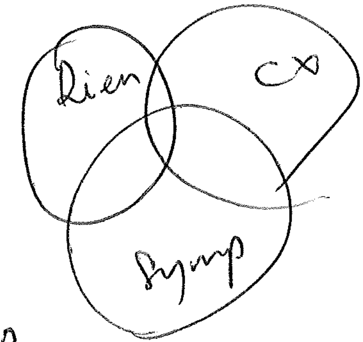
868 UMD . Yanir Rubinstein .

Outline: 1) Intro to Cx Geom.

(a) (almost) Cx structure (see diff).
Integrability.

(b) Kähler mflds. diff. characterisations.



2) Make friends with some Kähler mflds.

3) Fundamentals.

Ref: §2.1 in Rubinstein 2014
§2.13 p30.

- Understand mfld from choice of functions that respect structure.

Top mflds $\to$ C^0

C^∞ mflds $\to$ C^∞ (vastly smaller)

$\mathbb{C}$ mflds $\to$ Hol. (again smaller)

~~Def~~ ~~Almost~~ (Complex structure).

M C^∞ mfld, TM tangent bundle,

J endomorphism of TM $\Leftarrow$ Real ~~tangent~~ tangent bundle.

$J^2 = J \circ J = -I$.

(~~Almost~~ $\mathbb{C}$X) $\exists$ atlas of holomorphic charts.

~~Almost~~ $\xrightarrow{\text{~~Comp~~}}$ ~~$\exists J$~~.

i.e. $J\left(\frac{\partial}{\partial z_i}\right) = \sqrt{-1}\,\frac{\partial}{\partial z_i}$ } $\circ$?

Q. When does J induce a complex structure?

wrt f?

i.e.,

Type $(1,0)$: $\operatorname{span}\left\{\frac{\partial}{\partial z_i}\right\} = T^{(1,0)}M$.

Type $(0,1)$: $\operatorname{span}\left\{\frac{\partial}{\partial \bar z_j}\right\} = T^{0,1}M$.

$\boxed{[T^{1,0}, T^{1,0}] \subset T^{1,0}}$ (C)

Ex 1

$J^2 = -I \Rightarrow$ eigenvalues $= -i, +i$

$T^{1,0}$ = eigenspace of $+i$, $\bar T^{1,0}$ = eigenspace of $-i$.

From Ex 1, it is not clear that (C) holds, but for coordinate def^n, it is.

Nec. condition: for J to "come" from ex coord is (C).

Can phrase (C) as a natural eq^n:

$$T^{1,0} = \ker\left(I + \sqrt{-1}\, J\right). \quad \left(\begin{array}{l} v - \sqrt{-1}\, J v = 0 \\ -\sqrt{-1}\, v + J v = 0 \end{array} \right).$$

$X \in TM$ (Remember, TM is $\mathbb{R}$).

$\Downarrow$

$$\left.\begin{array}{l} X - iJX \\ Y - iJY \end{array}\right\} \in T^{1,0}.$$

$$\boxed{(I + iJ)\,[X - iJX,\ Y - iJY] = 0} \quad \text{(CC)}.$$

Prove. (C) $\Leftrightarrow$ (CC). $\forall X, Y$.

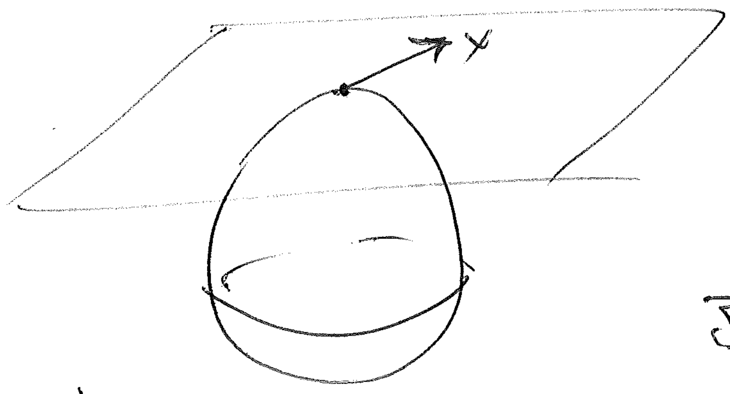
$$N_J(X, Y) := \cancel{[X - iJX,\ Y - iJY]}.$$
$$\mathrm{Re}\,(I + iJ).\,[X - iJX,\ Y - iJY].$$

Ex 2. Check N_J is a $(1,2)$ type

~~$N_J = 0$~~

Th$\underline{{}^{\text{m}}}$. $N_J = 0$ iff J Cx.

Example (1) S^2, g = round.



J T_xS ⟲.

$J|_x y = x \times y$.

(Almost Cx) 1). $x \cdot (x \times y) = 0$.

2) $x \times (x \times y) = -y$. (right hand rule).

~~Later~~ (Cx) . Let (z, U_N) ~~[illegible]~~ coords near n.p., ~~[illegible]~~ ~~[illegible]~~, and (w, U_S) near south pole.

$z = \frac{1}{w}$. on $U_N \cap U_S$.

(2). S^6 explicitly construct a non Cx ~~[illegible]~~ structure.
(Kirchoff 47).

Consider $S^6 \subset \mathbb{R}^7$.

" unit imaginary octonions

$\mathbb{O} \ni t = u_1 + u_2 i + u_3 j + u_4 k +$

$(v_1 + v_2 i + v_3 j + v_4 k) = u + v\ell.$

$u, v \in \mathbb{H}.$

$$\begin{cases} i^2 = j^2 = k^2 = \ell^2 = -1. \\ ij = k = -ji. \\ jk = -kj = i. \end{cases}$$

$\bar{t} = \cancel{u - v\ell} \; u_1 - u_2 i - u_3 j - u_3 k - v\ell.$

$J_t w = tw$, $\mathbb{O}$ multiplication.

① $w \in T_t S^6$

$\Uparrow$

$\operatorname{Im} \mathbb{O} \cap \underbrace{\ker(t\cdot)}_{\text{Scalar product kernel}}$.

$t = u + v\ell$, $w = x + y\ell$.

$\operatorname{Re}(tw) = \underset{\mathbb{R}^8}{(u,v) \cdot (x,y)} = 0$. (ie $\in \operatorname{Im} \mathbb{O}$).

Ex4. $tw \in \ker(t\cdot)$.

② $t(tw) = -w$.

Th (Kirchoff) $N_J \neq 0$.

Thw. (Kirchhoff.). No. #9.