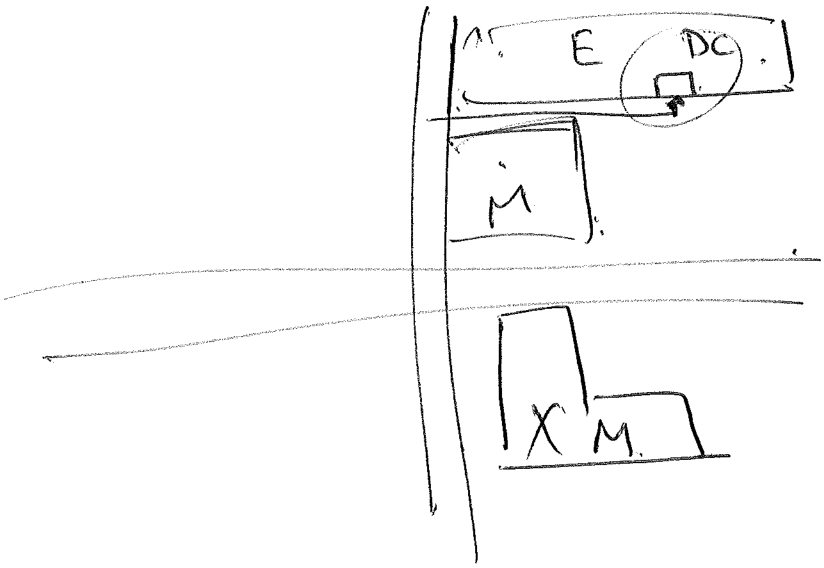
Lecture 1. Geometric Multivector Analysis. 01/09/2014.

Book - ~~e~~ technology building.



* Monday 13-15 ?
* Thursday 10-12.

Goal: M n-dim mfld., cpt.

① $\chi(M) = \operatorname{Ind}(\mathbb{D} : \wedge^{ev} M \to \wedge^{od} M) = \left(-\frac{1}{2\pi}\right)^{n/2} \int_M \langle Pf(R), d\hat{x} \rangle$.

(Chern-Gauss-Bonnet). (2 | n).

② $\operatorname{Ind}(\not{\mathbb{D}} : \not\triangle M \to \not\triangle M) = \left(\frac{i}{2\pi}\right)^{n/2} \int_M \langle \tilde{A}(R), d\hat{x} \rangle$

(Atiyah-Singer). (4 | n, oriented).

$Pf(R)$ - Pfaffian, $\underline{R}$ curvature tensor.

$\tilde{A}(R)$ - $\hat{A}$-roof functional.

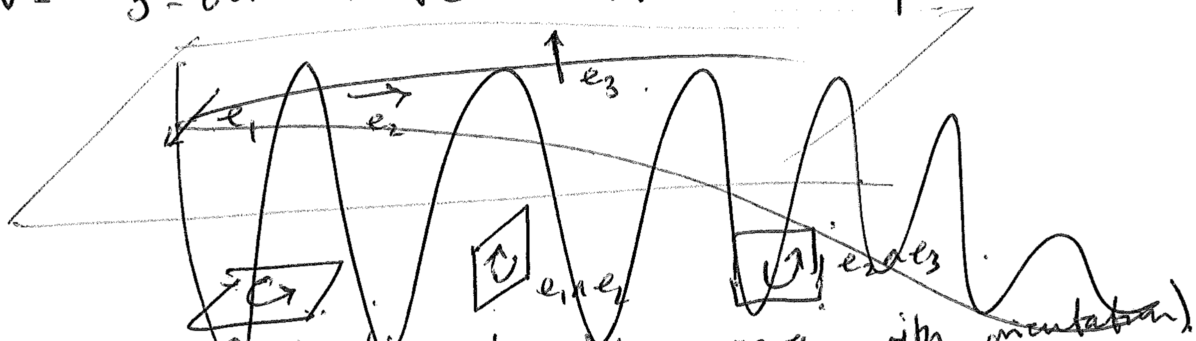
$\chi(M)$ - Euler characteristic.

$\mathbb{D}$

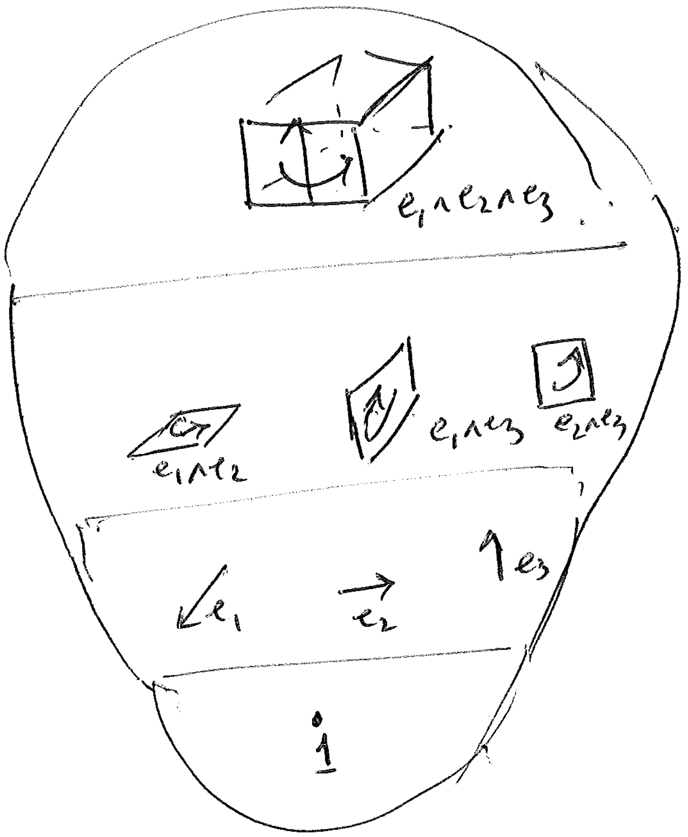
$\mathbb{D}$ - Clifford / Hodge-Dirac operator.

$\not{D}$ - Spin / Atiyah-Singer Dirac.

$V =$ 3-dim. ~~vector~~ Euclidean Space.



$e_1 \wedge e_2$ (bivector, 2dim area with orientation)



1	$\binom{3}{3}$
3	$\binom{3}{2}$
3	$\binom{3}{1}$
1	$\binom{3}{0}$

$e_1 \wedge e_2$ bivector, 2dim. area with orientation.

write $\wedge V$, in this case $\dim(\wedge V) = 8$.

In the integrals, $d\hat{x}$ = oriented n-volume element,

$Pf(R)$, $\hat{A}(R)$ = n-vector fields.

$\Lambda^{ev} M$, $\Lambda^{od} M$, from grading.

<u>Algebra</u>

exterior product $w_1 \wedge w_2$.

interior product.

$w_1 \lrcorner w_2$, $w_1 \llcorner w_2$.

Clifford product.

$w_1 \Delta w_2$.

<u>Geometry</u>

$V_1 \oplus V_2$.

$V_2 \ominus V_1 = V_2 \cap V_1^{\perp}$.

?

* case of two vectors.

$\langle v_1, v_2 \rangle$, $v_1 \wedge v_2$.

scalar, vector.

$$\boxed{v_1 \Delta v_2 = \underbrace{\langle v_1, v_2 \rangle}_{\substack{\text{maximize} \\ v_1 \parallel v_2}} + \underbrace{v_1 \wedge v_2}_{\substack{\text{maximized} \\ v_1 \perp v_2}}}$$

Two associative algebras. $(\Lambda V, \wedge)$, $(\Lambda V, \Delta)$.
$\Lambda V = (\Delta V)$

More invertible elements than ΛV.

Actually, $(\Delta V, \Delta)$ is a <u>matrix algebra</u>!

So, it acts on something $\rightsquigarrow$ Spinor Space $\cancel{\Lambda V}$.

"$\sqrt{\Delta V}$".

W = linear operators on ψ

$\Delta^+ V$ ψ

$\Delta^- V$ φ

$\not\Delta V$, dim $= 2^{\lfloor n/2 \rfloor}$.

(i.e., $n=3$, dim $\not\Delta V = 2$).
So, ΔV = 2-matrices,
$\not\Delta V$ = 2 vectors,
ΔV built from Pauli matrices.

Splits naturally into + and −, which corresponds to the fact that $\not\Delta V$ is an ext. algebra itself.

Differential Operators

Function $f(x)$, take $\partial_{x_i} f(x)$, corresponds vector e_i.

$$\nabla \wedge f(x) = \sum_{i=1}^{n} e_i \wedge \partial_{x_i} f(x) =: df \text{ (Ext der)}.$$

$$\nabla \lrcorner f(x) := \sum_{i=1}^{n} e_i \lrcorner \partial_{x_i} f(x) =: \delta f \text{ (Int der)}$$

$$\nabla \Delta f(x) = \sum e_i \Delta \partial_{x_i} f(x) = Df \text{ (Cliff der)}$$

$$\nabla \cdot \psi = \sum e_i \cdot \partial_{x_i} \psi(x) = D\psi \text{ (spin der)}.$$

product in $\not\Delta M$.

$\psi : M \to \not\Delta M$.

~~Rf~~ f function $\Rightarrow$ df usual derivative.

f 1-vector $\Rightarrow$ δf divergence.

For suitable choices of $\not{\partial}$, one can find ~~$\not{D}$ to be~~ Cauchy-Riemann eq^{ns} in $\not{D}$.