Lutz Weisss

01/10/2014.

(1). $\| (\sum_j |T_j x_j|^2)^{\frac{1}{2}} \|_{L^p} \le c \, \| (\sum_j |x_j|^2)^{\frac{1}{2}} \|_{L^p}$.

Marcinkiewicz - Zygmund:

$$\| (\sum_j |x_j|^2)^{\frac{1}{2}} \| \cong \mathbb{E} \| \sum_j r_j x_j \|$$

A has H^∞ F.C. on X B-space. meaning:

$$Ax = \sum_{k \in \mathbb{Z}} \varphi(2^k A) Ax . \quad \varphi \in H^\infty_0(\Sigma_\sigma)$$

$$\sum_k \varphi(2^k \lambda) = 1 ,$$

$$\|x\| \cong \mathbb{E} \| \sum_{k \in \mathbb{Z}} r_k \varphi(2^k A) x \|_X \leftarrow \text{B-space.}$$

$\Rightarrow$ Ask about how to go from Hilbert to Banach. Is this sharp? $\rightsquigarrow$ Lutz mentioned this in the middle of talk.

Lourdes Rodriguez Maria 01/10/2014.

Semigroup P_t. Markovian if

$P_t(1)$ is constant.

Q. This allows for good kernel estimates. But what is the role of constants here? In more general spaces, can this be replaced by something else, operator adapted?

This was associated to Laplacian.

So is $\text{const} \subset N(\Delta)$ the key?

Charles Batty 21/10/2014

Resolvent estimates to Decay rates.

$$\partial_t^2 u - \Delta u + 2a(x)\partial_t u = 0 \qquad t>0\ x\in\Omega$$

$$u(x,t) = 0. \qquad t>0\ x\in\partial\Omega$$

$$u(\cdot,0) = u_0 \in H^1(\Omega).$$

Ω cpt Reim mfld.

$a: \Omega \to [0,\infty)$ cts.

Energy $E(u,t) = \int |\nabla u|^2 + |\partial_t u|^2$.

a damping $\Rightarrow$ decreasing in t.

Known: $\{x : a(x) > 0\}$ satisfies geometric optics condition $\Rightarrow$ decay is exponential

Domain of damping.

$\underbrace{E(u,t) \to 0 \quad \text{as} \quad t \to \infty.}$

always true.

but without geo. optics cond, may be polynomial or logarithmic.

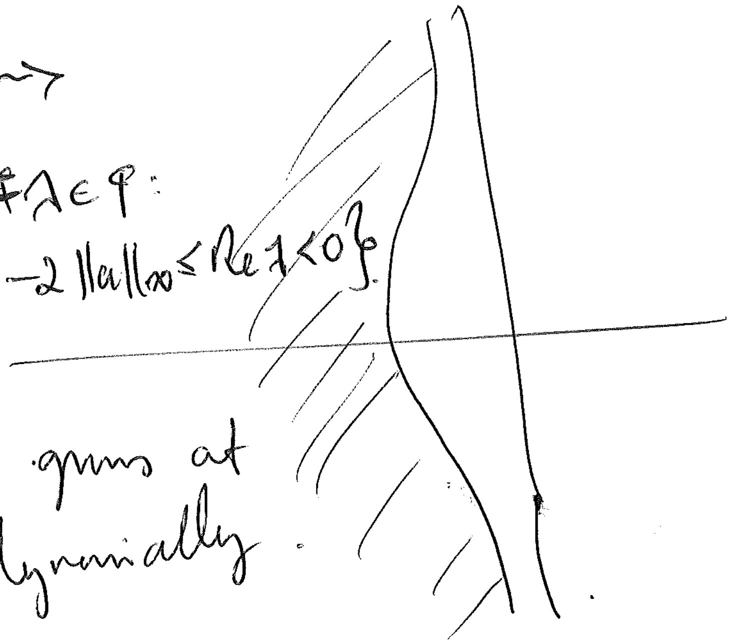
Reformulate: $X = H^1_0(\Omega) \times L^2(\Omega)$.

$$A = \begin{pmatrix} 0 & 1 \\ \Delta & -2a(x) \end{pmatrix}, \quad \mathcal{D}(A) = (H^2 \cap H^1_0) \times L^2.$$

$\rightsquigarrow$ $\underline{\mathcal{U}(t) = A\mathcal{U}'(t)}$.

Lebeau 96 $\rightsquigarrow$

$\sigma(A) \subset \{\lambda \in \mathbb{C} : -2\|a\|_\infty \le \mathrm{Re}\,\lambda < 0\}$.



$\|(is - A)^{-1}\|$ grows at most polynomially.

Note: Decay rate of $E(u,t)$ with initial data (u_0, u_1) corresponds to decay rate of

$$\|T(t)(\lambda - A)^{-1}\|$$

for $\lambda \in \rho(A)$.

Batty '88, showed $\|T(t)x\| \to 0$ as $t \to \infty$.

$M_{\log}$ - Thm of Batty-D. says that.

$$\|(is-A)^{-1}\| \leq M_{\log}(s).$$

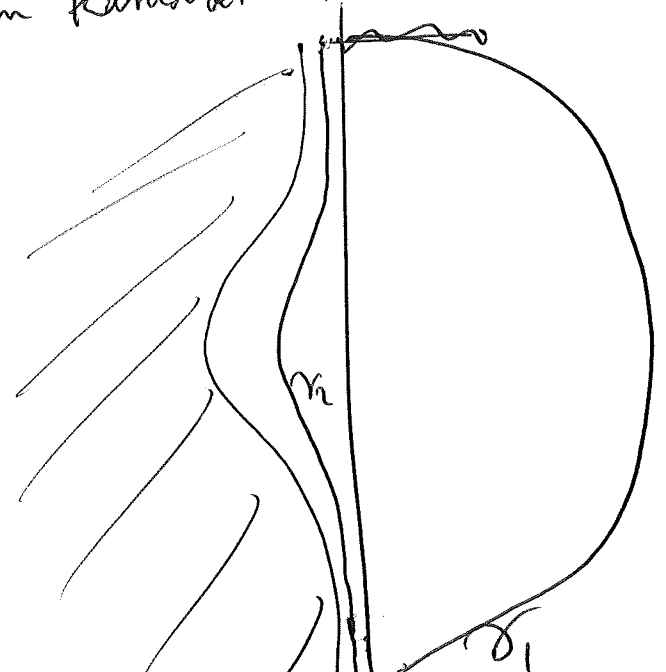
$$\Rightarrow \quad \|T(t)A^{-1}\| \leq \frac{1}{M^{-1}_{\log}(t)}.$$

where $M_{\log}$ includes logs.

The opposite is also true, but $M_{\log}$ replaced by a different (better one).

Pf — Ingham Karamata Thm generalisation to C_0-semigroups.

Carefully choose contour integral.



L^p norms for arclength being Carleson measure.

Need to choose contour γ_2 carefully to get best constants.

Q. Assumption of regularity on a? Drop continuity?

Yuri ~ Tomilov.

01/10/2014.

"Subordination · means"

$$\mu_t(\mathbb{R}^+) \le 1, \qquad \mu_t * \mu_s = \mu_{t+s} \;\ldots$$

Definition: ~~Idea:~~ encodes "semigroup",

$$e^{-t\varphi} = \int_0^\infty e^{-ts}\, d\mu_t(s).$$

Similar for ~~so~~ operators.

$(\mu_t)_{t\geq 0}$ subordinator.

$$T(t) = \int_0^\infty e^{-sA}\, \mu_t(ds).$$

Diego Chamorro 01/10/2014

- Weak Poincaré pseudo-inequalities:

$$\|q f - A_t S f\|_q \lesssim \alpha(t) \|\nabla f\|_1$$

What is this?

some factor in t.